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Effect of poloidal magnetic field on the propagation characteristics of ion Bernstein waves in the ion cyclotron resonance frequency range for tokamak

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Abstract · For radio frequency heating, the main requirement is that it should be possible to launch a wave from the plasma edge, get coupled to the plasma, propagate with minimum losses up to the core and then deposit the power at the required position in the plasma corresponding to a cyclotron resonance layer.

The absorption and propagation of the waves for radio-frequency heating in the ion cyclotron range of frequencies have been investigated using kinetic theory to take into account the effect of finite Larmor radius. In general, the dispersion relation contains two modes, one corresponding to cold plasma wave and another one corresponding to ion Bernstein wave. Sy, W N-C *et al* [1] have developed a dispersion relation for tokamak plasmas taking into account the two cold plasma waves and the ion Bernstein wave. However, they have neglected the effect of poloidal magnetic field on the propagation characteristics.

Here, we report the effect of poloidal magnetic field of the tokamak on the propagation characteristics The dispersion relation has been developed using two cold plasma waves and the ion Bernstein wave. It is expected that the effect of poloidal magnetic field is essential for the tokamak geometry, to get better heating in tokamak plasmas.

Keywords Ion Bernstein waves, radio frequency heating, tokamak.

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1. Introduction

Radio frequency heating involves launching into plasma high power electromagnetic waves, tuned to some natural resonant frequency of the plasma, which will lead to absorption of the wave and transfer of its energy to the plasma particles. For radio frequency heating, the main requirements are that it should be possible to launch a wave from an antenna or wave guide at the plasma edge and that the wave can propagate into the central region of the plasma and can be absorbed there.

The ion cyclotron frequency range of few tens of MHz is widely used in small as well as large tokamaks like JET (Joint European Torus) machine. In the ion cyclotron range of frequency, high power is available at reasonably low cost, conventional transmission lines can be used to carry power to the plasma, simple antenna can be used efficiently to couple the power to the plasma.

In the range of ion cyclotron resonance frequencies, both cold plasma electromagnetic waves *i.e.* slow waves and fast waves can be used to heat the plasma in which the power is deposited at the layer corresponding to the cyclotron frequency. Recently, a new method corresponding to the warm plasma mode called ion Bernstein wave heating (IBWH) has been developed and seems to be a highly efficient way of heating plasma in the range of ion cyclotron resonance frequencies.

2. Theory

A general theory of the plasma waves, with finite temperature effect and spatial gradient effects is known to be complicated, so we have derived a matrix for cold plasma waves and we have included the finite temperature modification term with some approximations. For the theoretical derivation, the Maxwell's equations in SI units are

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B} / \partial t \tag{2.1}$$

and

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + (1/c^2) \partial \boldsymbol{E} / \partial t, \tag{2.2}$$

where $\boldsymbol{J}$ is the induced current density. Consider the equation of motion for α-species (electrons, ions, helium) in SI units

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$$m_\alpha n_\alpha \partial V_\alpha / \partial t = q_\alpha n_\alpha [E + V_\alpha \times B], \tag{2.3}$$

where

m_α = mass of α-species,

n_α = density α-species,

V_α = velocity of α-species,

E = electric field,

k = propagation vector,

B_T = toroidal magnetic field along z-axis,

B = resultant magnetic field,

for the plane wave $\partial / \partial = -i\omega$. (2.4)

In the cylindrical coordinate system

$$x = \rho \cos \psi,\ y = \rho \sin \psi,\ z = z, \tag{2.5}$$

where ρ is the radius of the cylinder and ψ is the angle from x-axis in x–y plane. Thus,

$$B = B_p(\rho) \cos \psi\, e_x + B_p(\rho) \sin \psi\, e_y + B_T e_z, \tag{2.6}$$

where B_p poloidal magnetic field in (x–y) plane.

Separating the components of eq. (2.3) we get

$$V_{\alpha x} - R V_{\alpha y} B_T + R V_{\alpha z} B_p \sin \psi = R E_x, \tag{2.7}$$

$$R V_{\alpha x} B_T + V_{\alpha y} - R V_{\alpha z} B_p \cos \psi = R E_y, \tag{2.8}$$

$$-R V_{\alpha x} B_T \sin \psi + R V_{\alpha y} B_p \cos \psi + V_{\alpha z} = R E_z \tag{2.9}$$

where $R = iq\alpha / (m_\alpha \omega)$ (2.10)

solving (2.7), (2.8) and (2.9) by Cramer's rule, we get

$$V_{\alpha x} = (R / D_1)[E_x(1 + R^2 B_p^2 \cos^2 \psi + R B_T (E_y + R B_p \cos \psi E_z) + R B_p \sin \psi (R B_p \cos \psi E_y - E_z)], \tag{2.11}$$

$$V_{\alpha y} = (R / D_1)[(E_y + R B_p \cos \psi E_z) - E_x (R B_T - R^2 B_p^2 \sin \psi \cos \psi) + R B_p \sin \psi (R B_T E_z + R B_p \sin \psi E_y)], \tag{2.12}$$

$$V_{\alpha z} = (R / D_1)[(E_z - R B_p \cos \psi E_y) + R B_T (R B_T E_z + R B_p \sin \psi E_y) + E_x (R^2 B_p B_T \cos \psi + R B_p \sin \psi)], \tag{2.13}$$

where

$$D_1 = [(1 + R^2 B_p^2 \cos^2 \psi) + R B_T (R B_T - R^2 B_p^2 \sin \psi \cos \psi) + R B_p \sin \psi (R^2 B_p B_T \cos \psi + R B_p \sin \psi)] \tag{2.14}$$

The electric displacement D includes the vacuum displacement and the plasma current according to the relation

$$D = K \cdot E$$

$$= E + \{i / (\varepsilon_0 \omega)\}\, J, \tag{2.15}$$

where K is the dielectric tensor and

$$J = \textstyle\sum_\alpha n_\alpha q_\alpha V_\alpha . \tag{2.16}$$

The components of eq. (2.15) with the eq. (2.11), (2.12), (2.13) and (2.16) we get,

$$\begin{aligned} K_{xx} E_x &= A_{11} E_x - A_{12} E_y + A_{13} E_z, \\ K_{yy} E_y &= -B_{11} E_x + B_{12} E_y - B_{13} E_z, \\ K_{zz} E_z &= C_{11} E_x - C_{12} E_y + C_{13} E_z, \end{aligned} \tag{2.17}$$

where

$$\begin{aligned}
A_{11} &= 1 - \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(1 + R^2 B_p^2 \cos^2 \psi\right)\right], \\
A_{12} &= \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R B_r + R^2 B_p^2 \sin \psi \cos \psi\right)\right], \\
A_{13} &= \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R B_p \sin \psi - R^2 B_T B_p \cos \psi\right)\right], \\
B_{11} &= \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R^2 B_p^2 \sin \psi \cos \psi - R B_T\right)\right], \\
B_{12} &= 1 - \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(1 + R^2 B_p^2 \sin \psi\right)\right], \\
B_{13} &= \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R B_p \cos \psi + R^2 B_T B_p \sin \psi\right)\right], \\
C_{11} &= -\textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R^2 B_p B_T \cos \psi + R B_p \sin \psi\right)\right], \\
C_{12} &= \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(R^2 B_p B_T \sin \psi - R B_p \cos \psi\right)\right], \\
C_{13} &= 1 - \textstyle\sum_\alpha \left[\{\omega_{P\alpha}^2 / (\omega^2 D_1)\}\left(1 + R^2 B^2{}_T\right)\right],
\end{aligned} \tag{2.18}$$

$$\omega_{p\alpha}^2 = n_0 q_\alpha^2 / (\varepsilon_0 m_\alpha), \text{ (plasma frequency of } \alpha\text{-species).} \tag{2.19}$$

The eq. (2.17) can be written as

$$D = \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -B_{11} & B_{12} & -B_{13} \\ C_{11} & -C_{12} & C_{13} \end{bmatrix} E_z \tag{2.20}$$

since we know that

$$n \times (n \times E) + D = 0, \tag{2.21}$$

where $n = ck/\omega$ (2.22)

is a dimensionless vector, called refractive index.

Thus the combined eqs. (2.20) and (2.21) give

$$\begin{bmatrix} A_{11} - (n_y^2 + n_z^2) & -A_{12} + n_x n_y & A_{13} + n_x n_z \\ -B_{11} + n_x n_y & B_{12} - (n_z^2 + n_x^2) & -B_{13} + n_y n_z \\ C_{11} + n_x n_z & -C_{12} + n_y n_z & C_{13} - (n_x^2 + n_y^2) \end{bmatrix} \begin{bmatrix} E_x \\ \\ E_z \end{bmatrix} = 0. \tag{2.23}$$

Now the term arising from finite temperature modification is given by

$$\tau = \textstyle\sum_\alpha 3 \omega_{p\alpha}^2 T_\alpha / [m_\alpha (\omega^2 - 4\omega_{c\alpha}^2)(\omega^2 - \omega_{c\alpha}^2)]^{(3)}, \tag{2.24}$$

where

$\omega_{c\alpha}$ = cyclotron frequency of α-species,

T_α = temperature of α-species.

Now introducing the dimensionless variable

$$\xi = x\omega / c \text{ and } T = -\tau\omega^2 / c^2, \tag{2.25}$$

the eq. (2.23) with eq. (2.24) becomes

$$\begin{matrix} A_{11}+\tau\partial_x^2-(n_y^2+n_z^2) & -A_{12}+n_x n_y & A_{13}+n_x n_z \\ -B_{11}+n_x n_y & B_{12}-(n_z^2+n_x^2) & -B_{13}+n_y n_z \\ C_{11}+n_x n_z & -C_{12}+n_y n_z & C_{13}-(n_x^2+n_y^2) \end{matrix}$$

$$'y \quad 0, \quad (2.26)$$
$$\lfloor E_z \rfloor$$

where $\tau\partial_x^2$ is defined by eqs. (2.24) and (2.25). For the plasma with inhomogeneity in the x-direction using

$$n_x = -\iota d/d\xi \quad (2.27)$$

since

$$n_x = ck_x/\omega \quad (2.28)$$

and $\quad \partial/\partial_x = \iota k_x, \quad (2.29)$

the non-trivial solution of eq. (2.26) with (2.27) provides a set of three differential equations

$$-T(d^2E_x/d\xi^2)+D_{11}E_x-A_{12}E_y-\iota n_y(dE_y/d\xi)$$
$$+A_{13}E_z-\iota n_z(dE_z/d\xi)=0, \quad (2.30)$$

$$-B_{11}E_x-\iota n_y(dE_x/d\xi)+E_{11}E_y+(d^2E_y/d\xi^2)+L_{11}E_z=0, \quad (2.31)$$

$$C_{11}E_x-\iota n_z(dE_x/d\xi)+F_{11}E_y+M_{11}E_z+(d^2E_z/d\xi^2)=0. \quad (2.32)$$

Multiplying eq. (2.30) by $-i$ we get

$$\iota T(d^2E_x/d\xi^2)-\iota D_{11}E_x+\iota A_{12}E_y-n_y(dE_y/d\xi)$$
$$-\iota A_{13}E_z-n_z(dE_z/d\xi)=0, \quad (2.33)$$

where

$$\begin{aligned} D_{11} &= A_{11}-(n_y^2+n_z^2), \\ E_{11} &= B_{12}-n_z^2, \\ F_{11} &= -C_{12}+n_y n_z, \\ L_{11} &= -B_{13}+n_y n_z, \\ M_{11} &= C_{13}-n_y^2. \end{aligned} \quad (2.34)$$

Solving eqs. (2.30), (2.31) and (2.32) we get the polynomial

$$-\iota TD^8+0D^7+FF_{11}D^6+FF_{22}D^5+FF_{33}D^4+FF_{44}D^3$$
$$+FF_{55}D_2+FF_{66}D+FF_{77}=0 \quad (2.35)$$

where

$D=d/d\xi=\iota n_x$

$$FF_{11}=\iota TE_{444}-E_{111}+\iota n_y n_z,$$

$$FF_{22}=-(E_{222}+n_z A_{13}+n_z C_{11}),$$

$$FF_{33}=\iota TE_{555}-E_{333}+E_{111}E_{444}+\iota n_z E_{888}-n_z E_{666}-\iota A_{13}C_{11},$$

$$FF_{44}=E_{222}E_{444}+\iota n_z E_{999}-C_{11}E_{888}-\iota A_{13}E_{666}-n_z E_{777},$$

$$FF_{55}=E_{111}E_{555}+E_{333}E_{444}-C_{11}E_{999}-E_{666}E_{888}-\iota A_{13}E_{777},$$

$$FF_{66}=E_{222}E_{555}-E_{666}E_{999}-E_{777}E_{888},$$

$$FF_{77}=E_{333}E_{555}-E_{777}E_{999},$$

$$\begin{aligned} E_{111} &= \iota TE_{11}-\iota D_{11}-\iota n_y^2, \\ E_{222} &= -(A_{12}+B_{11})n_y, \\ E_{333} &= \iota(-D_{11}E_{11}+A_{12}B_{11}), \\ E_{444} &= -(M_{11}+E_{11}), \\ E_{555} &= (L_{11}F_{11}-M_{11}E_{11}), \\ E_{666} &= \iota(n_y F_{11}-n_z E_{11}), \\ E_{777} &= (B_{11}F_{11}+C_{11}E_{11}), \\ E_{888} &= (n_z E_{11}-n_y L_{11}), \\ E_{999} &= \iota(A_{13}E_{11}+L_{11}A_{12}). \end{aligned}$$

Since the analytical solution of (2.35) is not possible, so we have solved the dispersion relation numerically for the parabolic density profile and squared parabolic temperature profile for the typical parameters of Aditya tokamak. The eight roots of the polynomial becomes approximately duplicate. Numerically we have seen out of four roots in n_x^2, one root matches with the Stix's standard notation S in some frequency range. So we have neglected the S root and we have done the analysis of the remaining three roots in n_x^2 which are slow wave, fast wave and ion Bernstein wave. The short and long wave lengths propagating branches represent ion Bernstein waves and fast waves respectively, while the evanescent branch represent slow waves. For the parameters of Aditya tokamak we have developed a computer code [4] for the roots, which are given by the Figures 1–3.

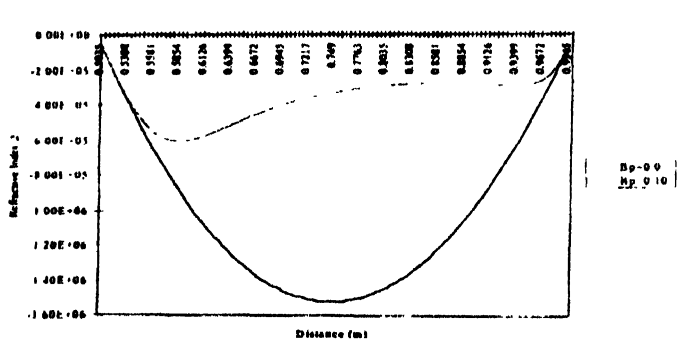

Figure 1. Dispersion curves for slow wave (square of the refractive index *versus* distance from inner end of the vessel) with parameters . major radius = 0.75 m, minor radius = 0.25 m, hydrogen plasma with 5% helium, maximum electron (ion) density = 2.25e19 m^{-3}, edge electron (ion) density = 1.0e16 m^{-3}, B_T = 1.5 T, B_p = 0.10 T, applied radio frequency at the centre of the device = 34.451584272e6, refractive index along y- and z-axis = 0.50, maximum electron (ion) temperature = 150.0 eV, edge electron (ion) temperature = 10.0 eV.

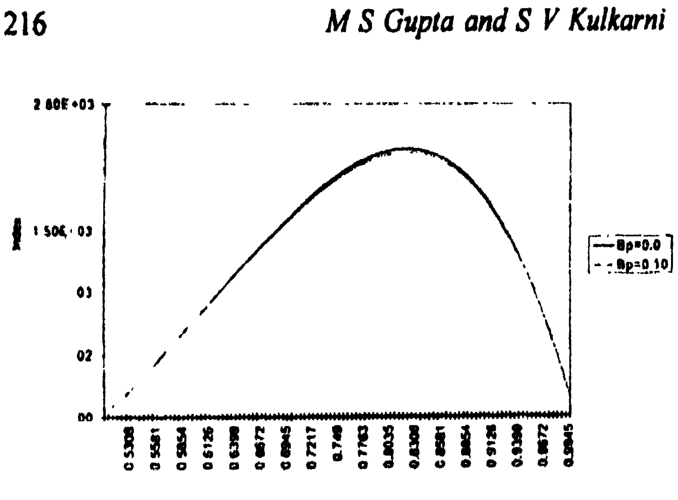

Figure 2. Dispersion curves for fast wave (square of the refractive index *versus* distance from inner end of the vessel) with parameters of Figure 1.

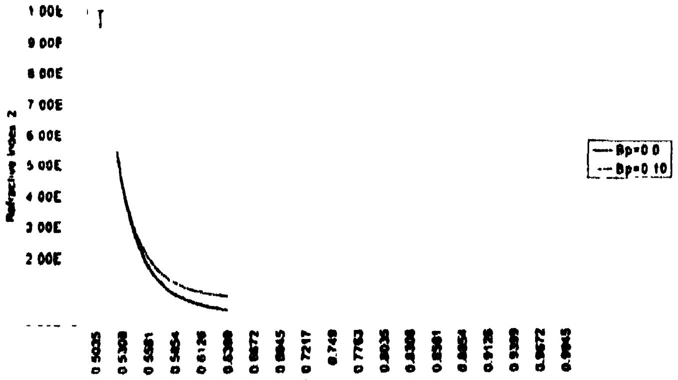

Figure 3. Dispersion curves for ion Bernstein waves (square of the refractive index *versus* distance from inner end of the vessel) with parameters of Figure 1.

3. Conclusions

The dispersion relation n^8 having 4 roots in n^2, out of which of one root matches with Stix's standard notation S, the remaining three roots in n^2 known as ion Bernstein waves, fast waves, slow waves. We have also calculated all the roots in presence of poloidal magnetic field. We have seen that the behaviour of the roots more realistic in presence of poloidal magnetic field.

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